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Two graviton production at e^+e^- and hadron hadron colliders in the Randall-Sundrum model

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Abstract: We compute the pair production cross section of two Kaluza Klein modes in the Randall-Sundrum model at e^+e^- and hadron hadron colliders. These processes are interesting because they get dominant contribution from the graviton interaction at next to leading order. Hence they provide a nontrivial test of the low scale gravity models. All the Feynman rules at next to leading order are also presented. These rules may be useful for many phenomenological applications including the computation of higher order loop corrections.

Keywords: Extra Large Dimensions, Models of Quantum Gravity.

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Contents

1. Introduction

The proposed existence of large extra dimensions [1, 2] might provide a solution to the hierarchy problem since it can considerably lower the scale of quantum gravity. In this model the standard model fields are assumed to be confined on a $3+1$ dimensional hypersurface or a brane in a higher dimensional space-time [3]. In this low scale gravity model, which is generally refered to as the ADD model, gravity may become strong even at TeV scale which is far below the Planck scale 10^{19} GeV. The two scales are related via gauss law given by $M_{pl}^2 = M_D^{2+n} V_n$, where V_n is volume of n extra dimensional space. If we take $M_D \sim$ TeV then the size of the extra dimension lies in the range 1 mm–10 fm for $n = 2$–6. However by doing so we introduce a new hierarchy between the compactification scale and the fundamental scale in the theory. An alternative low scale gravity model (the RS model) [4] involves the existence of a single extra dimension in a warped geometry. In this case the scale of quantum gravity can be as small as 1 TeV without requiring the existence of very large extra dimensions. These low scale gravity models are reviewed in ref. [5, 6]. The leading order Feynman rules for the ADD model are given in Lykken et al [7] and Giudice et al [8].

Phenomenologically the RS model [9] is very different from ADD model because of the presence of very massive KK modes which produce observable signatures in a collider experiment. In the case of ADD model we instead have large number closely spaced KK modes with the mass of the lightest mode governed by the size of the large extra dimension.

These modes cannot be observed directly due to their small coupling to matter and hence the production of these modes gives rise to missing energy signatures. However in the RS model the first few modes produced can decay into observable particles. The center of mass energy required to produce these resonances are well within the energies available in the next generation of colliders such as the LHC, NLC. Non-observation of these processes in experiments constrain the parameters of this theory. Several phenomenological studies have been carried out to study these resonances in e^+e^- colliders [10]–[13] and in hadron colliders [14]–[18].

In the present paper we study the process of two graviton production in the RS model in e^+e^- and hadron hadron colliders. This process gets contribution at next to leading order in the gravitational coupling. Hence we derive the required Feynman rules at this order. This process may not be very important in case of ADD model because these modes just escape through the detector undetected. However our calculation can be easily generalized for this case also.

The plan of the rest of the paper is as follows. In the section 2 we briefly review the derivation of lowest order interaction lagrangian. We then extend the calculation to next order in coupling and find the corresponding interaction lagrangian. Section 3 describes all processes related to emission of two lowest massive graviton modes. Here we also present our results for the cross section of two graviton emission at e^+e^- and hadron hadron colliders. Section 4 gives our conclusions. In appendix A we present all the Feynman rules involving vertex of matter fields with two gravitons at next to leading order in coupling. In appendix B, we derive the Feynman rule for three graviton vertex which also contributes to the process under consideration.

2. Interaction lagrangian

The RS model assumes a five dimensional non-factorizable geometry where two $3-branes$, with opposite tensions, reside at $S^1/\mathbb{Z}_2$ orbifold fixed points. In this framework solution to 5-dimensional Einstein equations is given by

$$ds^2 = e^{-\sigma(\phi)}\eta_{\mu\nu}dx^\mu dx^\nu + r_c^2 d\phi^2 \,, \tag{2.1}$$

where ϕ is the angular coordinate parameterizing the extra dimension, $\sigma(\phi) = kr_c|\phi|$ with r_c being the compactification radius of the extra dimension, and $0 \leq |\phi| \leq \pi$. Starting from the 5-D action, after integrating over extra coordinate ϕ, we obtain the reduced effective 4-D planck scale given by the relation

$$M_{Pl}^2 = \frac{M^3}{k}(1 - e^{-2kr_c\pi}) \,. \tag{2.2}$$

Setting visible brane at $\phi = \pi$, where our standard model fields live, it is found that any mass parameter $\tilde{m}$ on the visible 3-brane in the fundamental higher-dimensional theory will correspond to a physical mass

$$m = e^{-kr_c\pi}\tilde{m} \,.$$

Hence TeV mass scales can be generated on the 3-brane at $\phi = \pi$ due to exponential factor present in the metric if we assume $kr_c \approx 12$.

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A small perturbation $h_{\mu\nu}$ of the metric, such that $e^{-2\sigma(\phi)}\eta_{\mu\nu} \to e^{-2\sigma(\phi)}(\eta_{\mu\nu} + h_{\mu\nu})$, can be expanded as,

$$h_{\mu\nu}(x,\phi) = \sum_{n=0}^{\infty} h_{\mu\nu}^{(n)} \frac{\chi^{(n)}(\phi)}{\sqrt{r_c}}\,. \tag{2.3}$$

After solving the linearised equation of motion of graviton field and working in gauge

$$\partial^{\mu} h_{\mu\rho}^{(n)} = h_{\mu}^{\mu(n)} = 0\,,$$

one obtains the solution for $\chi^{(n)}(\phi)$,

$$\chi^{(n)}(\phi) = \frac{e^{2\sigma(\phi)}}{N_n} J_2\left(\frac{M_n}{k} e^{\sigma(\phi)}\right) + \alpha_n Y_2\left(\frac{M_n}{k} e^{\sigma(\phi)}\right). \tag{2.4}$$

The exact expression for α_n is given in ref. [9]. The masses of the KK modes, M_n, are given by [9, 19]

$$M_n = x_n k e^{-kr_c\pi}\,, \tag{2.5}$$

where x_ns are the solution of the equation $J_1(x_n) = 0$. We also define the mass parameter $m_0 = M_n/x_n$. As we see from eq. (2.5), masses of the graviton KK excitations are dependent on the roots of J_1 and are not equally spaced. In the following, we formulate the interaction of physical KK modes to the Standard Model fields which reside on the brane at $\phi = \pi$.

To start with, let us first consider the minimal gravitational coupling of the general scalar S, vector V and fermion F,

$$\int d^4x \sqrt{-\hat{g}}\mathcal{L}(\hat{g}, S, V, F)\,, \tag{2.6}$$

where the induced metric $\hat{g}$ on the brane can be decomposed as,

$$\hat{g}_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}\,.$$

By expanding the interaction lagrangian upto $\mathcal{O}(\kappa^2)$, we get

$$\begin{aligned}\int d^4x \int d\phi \sqrt{-\hat{g}}\mathcal{L}(\hat{g}) = \int d^4x \int d\phi\,\delta(\phi-\pi)\Bigg[\left[\mathcal{L}(\hat{g})|_{\hat{g}=\eta} - \frac{\kappa}{2}h^{\mu\nu}T_{\mu\nu}\right] + \\ + \kappa^2\left[A\mathcal{L}(\hat{g})|_{\hat{g}=\eta} - B^{\mu\nu}\frac{\delta\mathcal{L}}{\delta\hat{g}^{\mu\nu}}|_{\hat{g}=\eta}\right] + \\ + \kappa^2 h^{\mu\nu}(x,\phi)\left[\frac{1}{2}\int d^4y \frac{\delta^2\mathcal{L}}{\delta\hat{g}^{\mu\nu}\delta\hat{g}^{\rho\sigma}}|_{\hat{g}=\eta} h^{\rho\sigma}(y,\phi)\right]\Bigg],\end{aligned} \tag{2.7}$$

where we have used

$$\hat{g}^{\mu\nu} = \eta^{\mu\nu} - \kappa h^{\mu\nu} + \kappa^2 h^{\mu\rho}h_{\rho}^{\nu} \tag{2.8}$$

$$\sqrt{-\hat{g}} = 1 + \frac{\kappa}{2}h + \kappa^2\left(\frac{1}{8}h^2 - \frac{1}{4}h_{\rho\sigma}h^{\rho\sigma}\right). \tag{2.9}$$

Several coefficients that appears in eq. (2.7) are given by,

$$A = \frac{1}{8}h^2 - \frac{1}{4}h_{\rho\sigma}h^{\rho\sigma}\,, \tag{2.10}$$

$$B^{\mu\nu} = \frac{1}{2}hh^{\mu\nu} - h^{\mu\lambda}h^{\nu}_{\lambda}\,, \tag{2.11}$$

$$h = h^{\mu}_{\mu} = h^{\mu\nu}\eta_{\mu\nu}\,. \tag{2.12}$$

Interaction lagrangian upto $\mathcal{O}(\kappa)$ is given by

$$-\frac{\kappa}{2}h^{\mu\nu}(x,\phi=\pi)T_{\mu\nu} = -\frac{1}{M^{3/2}}h^{\mu\nu}(x,\phi=\pi)T_{\mu\nu}\,,$$

where $\kappa = \frac{2}{M^{3/2}}$. With the help of eq. (2.2) we can express the above interaction lagrangian in terms of KK modes as,

$$-\frac{1}{M_{Pl}}h^{\mu\nu,0}T_{\mu\nu} - \frac{1}{\Lambda_\pi}\sum_{n=1}^{\infty}h^{\mu\nu,n}T_{\mu\nu}\,, \tag{2.13}$$

where $T_{\mu\nu}$, the energy-momentum tensor of the matter field confined on the brane, is given by,

$$T_{\mu\nu} = -\eta_{\mu\nu}\mathcal{L}(\hat{g})|_{\hat{g}=\eta} + 2\frac{\delta\mathcal{L}}{\delta\hat{g}^{\mu\nu}}|_{\hat{g}=\eta} \tag{2.14}$$

and

$$\Lambda_\pi = M_{Pl}e^{-kr_c\pi}\,.$$

Now we are interested in finding out the $\mathcal{O}(\kappa^2)$ Feynman rules for the gravitational interaction with matter confined on the D_3 brane embedded in the RS bulk.

The κ^2 part of the lagrangian that resembles the gravitational interactions with matter involving two gravitons, is given by

$$\begin{aligned}
\mathcal{L}_2 = \frac{4}{M^3}\Bigg[&\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}h^{\mu\lambda,n}\chi^n(\phi=\pi)h^{\nu,m}_{\lambda}\chi^m(\phi=\pi)\;\frac{\delta\mathcal{L}}{\delta\hat{g}^{\mu\nu}}\bigg|_{\hat{g}=\eta} - \\
&-\left(\frac{1}{2}\right)\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}h^{n}\chi^n(\phi=\pi)h^{\mu\nu,m}\chi^m(\phi=\pi)\;\frac{\delta\mathcal{L}}{\delta\hat{g}^{\mu\nu}}\bigg|_{\hat{g}=\eta} + \\
&+\left(\frac{1}{8}\right)\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}h^{n}\chi^n(\phi=\pi)h^{m}\chi^m(\phi=\pi)\;\mathcal{L}(\hat{g})|_{\hat{g}=\eta} - \\
&-\left(\frac{1}{4}\right)\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}h^{n}_{\rho\sigma}\chi^n(\phi=\pi)h^{\rho\sigma,m}\chi^m(\phi=\pi)\;\mathcal{L}(\hat{g})|_{\hat{g}=\eta} + \\
&+\left(\frac{1}{2}\right)\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}h^{\mu\nu,n}\chi^n(\phi=\pi)\;\frac{\delta^2\mathcal{L}}{\delta\hat{g}^{\mu\nu}\delta\hat{g}^{\rho\sigma}}\bigg|_{\hat{g}=\eta}h^{\rho\sigma,m}\chi^m(\phi=\pi)\Bigg]\,.
\end{aligned} \tag{2.15}$$

After substituting the expression of χ^n in the above equation and using the relation given in eq. (2.2), we finally get

$$\mathcal{L}_2 = \mathcal{L}_2^{00} + 2{\mathcal{L}_2}^{0n} + \mathcal{L}_2^{nm}\,, \tag{2.16}$$

where the $\mathcal{L}_2^{nm}$ part of the lagrangian is given by

$$\mathcal{L}_2^{nm} = \frac{4}{\Lambda_\pi^2}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\Bigg[h^{\mu\lambda,n}h_\lambda^{\nu,m}\,\frac{\delta\mathcal{L}}{\delta\hat{g}^{\mu\nu}}\bigg|_{\hat{g}=\eta} - \left(\frac{1}{2}\right)h^n h^{\mu\nu,m}\,\frac{\delta\mathcal{L}}{\delta\hat{g}^{\mu\nu}}\bigg|_{\hat{g}=\eta} + \left(\frac{1}{8}\right)h^n h^m\,\mathcal{L}(\hat{g})|_{\hat{g}=\eta} -$$
$$- \left(\frac{1}{4}\right)h^n_{\rho\sigma}h^{\rho\sigma,m}\,\mathcal{L}(\hat{g})|_{\hat{g}=\eta} + \left(\frac{1}{2}\right)h^{\mu\nu,n}\,\frac{\delta^2\mathcal{L}}{\delta\hat{g}^{\mu\nu}\delta\hat{g}^{\rho\sigma}}\bigg|_{\hat{g}=\eta}h^{\rho\sigma,m}\Bigg]. \tag{2.17}$$

The $\mathcal{L}_2^{00}$ part of the lagrangian is obtained by setting $n = m = 0$ and by replacing the coefficient Λ_π^2 by M_{Pl}^2 in $\mathcal{L}_2^{nm}$. Due to the large suppression factor $1/M_{Pl}^2$ the contribution of this lagrangian as well as that of ${\mathcal{L}_2}^{0n}$ is negligible. By using the interaction lagrangian, we can derive the feynman rules for two graviton interacting with matter fields which are given in appendix A of this paper. Also the feynman rules for three graviton vertex are presented in appendix B. The triple graviton vertex has also been studied in ref. [20].

3. Two graviton production

We next consider the production of two gravitons at e^+e^-, $p\bar{p}$ and pp colliders. We first consider the process fermion anti-fermion annihilation into two gravitons. There are four feynman diagrams contributing to this which are shown in figure 1. The differential cross section for $e^+e^- \to GG$ is given in eq. (C.1) in appendix C. The cross section also gets contribution from the s-channel diagram which involves a graviton propagator. This gets contribution from all the Kaluza-Klein modes. By summing over all the contributions the propagator takes the form

$$D(s) = \frac{32\pi c_0^2}{m_0^2}\sum_n \frac{1}{s - M_n^2 + i\Gamma_n M_n} = \frac{32\pi c_0^2}{m_0^4}\lambda_s(x_s)\,, \tag{3.1}$$

where $x_s = \frac{\sqrt{s}}{m_0}$ and $c_0 = k/M_{\rm pl}$. By taking into account all decay channels, Γ_n can be written as

$$\Gamma_n = c_0^2 m_0 x_n^3 \Delta(M_n)\,. \tag{3.2}$$

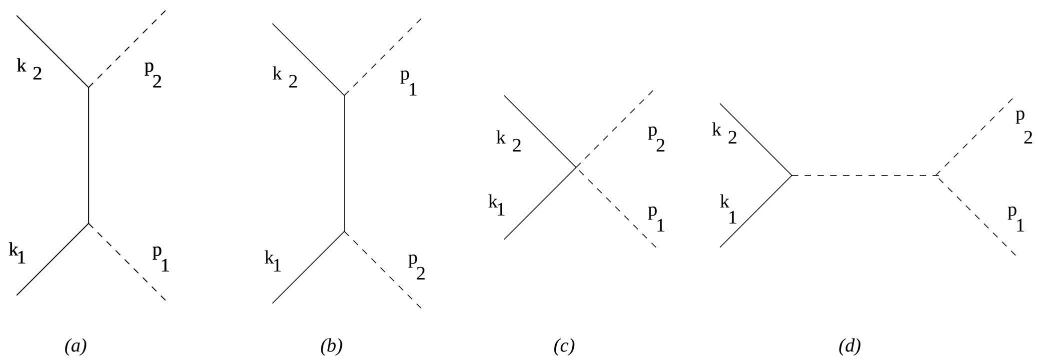

Figure 1: All possible tree level Feynman diagrams which contribute to two graviton emission due to fermion anti-fermion annihilation. The solid and dotted lines represent the fermions and the gravitons respectively.

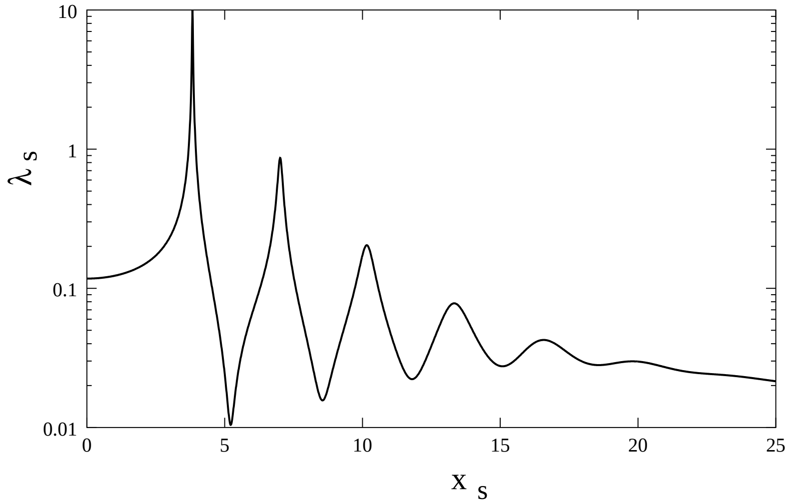

Figure 2: The graviton propagator factor λ_s as a function of $x_s = \sqrt{s}/m_0$. Here we have fixed the parameters $m_0 = 100\,\mathrm{GeV}$ and $c_0 = .01$.

Here Δ is a complicated function of M_n. We calculate the $\lambda_s(x_s)$ numerically and it's behaviour is shown in figure 2.

In figure 3 we show the s dependence of the production cross section of the two lightest KK modes of mass $M_1 = 3.83m_0$ in e^+e^- collisions. The resonance peaks which arise due to the s-channel graviton exchange (diagram 1d) are clearly visible in a certain range of parameter space. The peaks tend to disappear for large values of the parameter $c_0 = k/M_{\mathrm{pl}}$. We expect that the first few peaks will be identifiable if the parameter c_0 is sufficiently small. For values of c_0 larger than approximately 0.6 even the first resonance may not be clearly identifiable [13]. The maximum value of the parameter m_0 that can be explored with this process is given by $\sqrt{s} < 2M_1 = 7.66m_0$. In figure 4, we show the angular dependence of differential cross section for pair production of the first graviton mode with mass $M_1 = 3.83m_0$, where m_0 has been chosen to be $150\,\mathrm{GeV}$.

We see from figure 3 that, as expected, the cross section rises very rapidly with s. The perturbation theory is applicable only if $s \ll \Lambda_\pi^2$. In our calculations we have chosen the parameter space such that this condition is respected and perturbative predictions can be trusted. Perturbation theory breaks down for values of c_0 much larger than those chosen in figure 3 for the corresponding values of m_0. In this region it is not possible to compute this process reliably with our current understanding of quantum gravity.

The hadron-hadron crosss section gets contributions both from quarks and gluons. In the case of quark anti-quark annihilation the contributing diagrams are same as listed in figure 1. The diagrams contributing to gluon gluon fusion are also the same as those given in figure 1 with the quark lines replaced by gluons. The results for the pair production of the lightest KK mode with mass $M_1 = 3.83m_0$ is given in figures 5, 6 and 7. The center of mass energy $\sqrt{s}$ of the hadron hadron system is taken to be the LHC energy, i.e. $14\,\mathrm{TeV}$. In figure 5 we show the W^2 dependence of the cross section, where $W^2 = \tau s = x_1 x_2 s$ is the invariant mass of the final state and x_1 and x_2 are the longitudinal momentum fractions

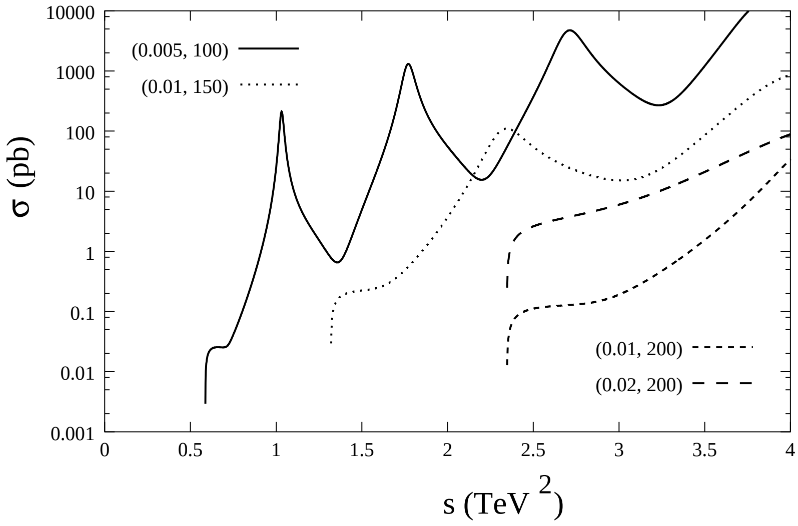

Figure 3: The two graviton production cross section in e^+e^- collisions as a function of the center of mass energy squared s (TeV^2) for several different values of the parameters (c_0, m_0), where m_0 is given in GeV.

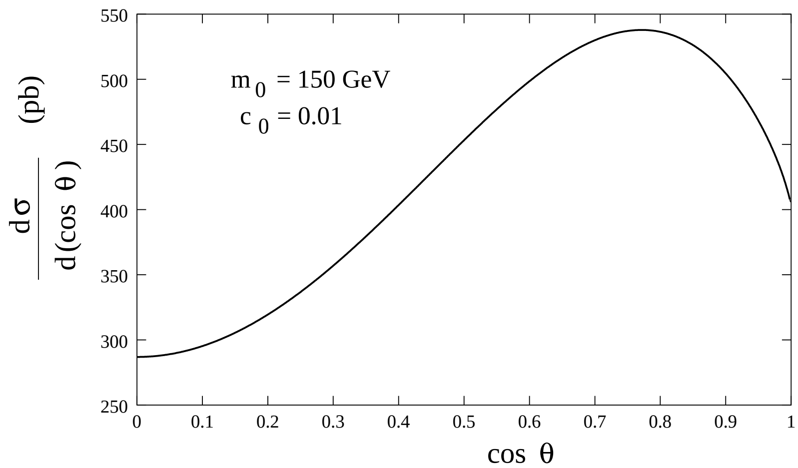

Figure 4: The differential cross section for $\sqrt{s} = 2$ TeV as a function of $\cos\theta$ for the production of the first KK mode in e^+e^- collisions. Here θ is the center of mass scattering angle and we have fixed $c_0 = 0.01$, $m_0 = 150\,\mathrm{GeV}$.

of the two initial state partons. We see from this figure that, as expected, LHC can probe a considerable range of parameters of the RS model through this process. In figures 6 and 7 we show the $\cos\theta$ and the rapidity $y = \frac{1}{2}\log(x_1/x_2)$ dependence of the cross section respectively. Here θ is the center of mass scattering angle in the parton subsystem. In figures 8, 9 and 10 we show the corresponding results for the proton anti-proton system at center of mass energy $\sqrt{s} = 2\,\mathrm{TeV}$, relevant for the Tevatron.

Each of the two gravitons produced in the final state eventually decay into either two jets, a lepton pair or a vector boson pair. Therefore one possible experimental signature of this process involves a four jet event such that the invariant mass of the two pairs of jets

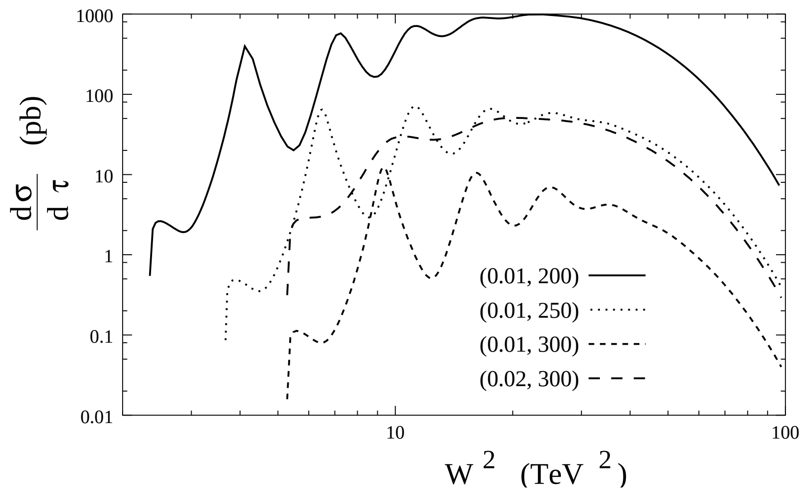

Figure 5: The invariant mass W^2 dependence of the two graviton production cross section in proton proton collisions at $\sqrt{s} = 14\,\text{TeV}$ for several different values of the parameters (c_0, m_0), where m_0 is given in GeV. Here $W^2 = \tau s$ is the invariant mass of the two graviton final state.

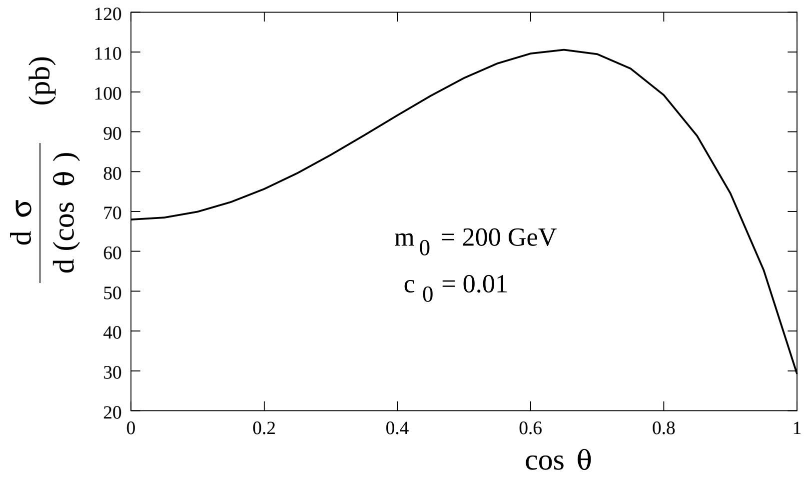

Figure 6: The $\cos\theta$ dependence of the two graviton production cross section in proton proton collisions at $\sqrt{s} = 14\,\text{TeV}$. Here θ is the center of mass scattering angle of the partonic subsystem.

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is equal to the mass of the graviton. There will be considerable QCD background which may limit the parameter space that can be explored with this process. We postpone the detailed study of experimental signatures to future research.

4. Conclusions

We have computed the pair production cross section of Kaluza Klein modes in the RS model in e^+e^- and hadron hadron colliders at leading order in the gravitational coupling κ. The leading order calculation requires the Feynman rules at order κ^2. This process displays a resonance structure due to the contribution from the s-channel KK mode exchange. For a large range of the parameter space this resonance structure is visible in the two graviton

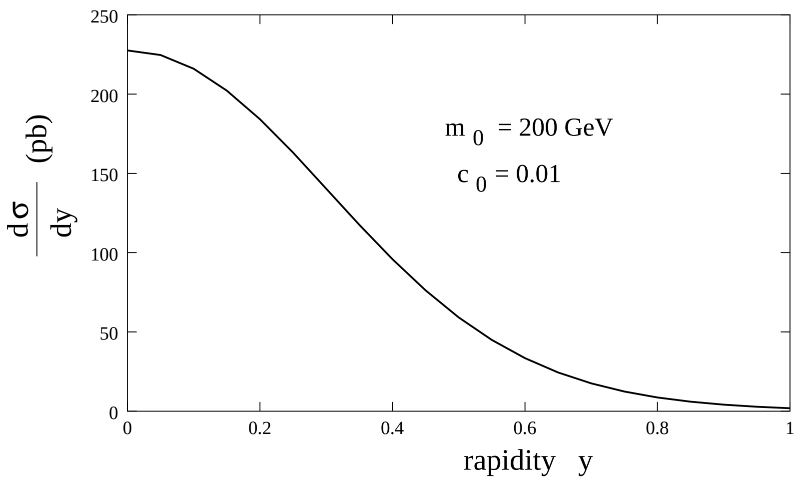

Figure 7: The rapidity dependence of the two graviton production cross section in proton proton collisions at $\sqrt{s} = 14\,\text{TeV}$.

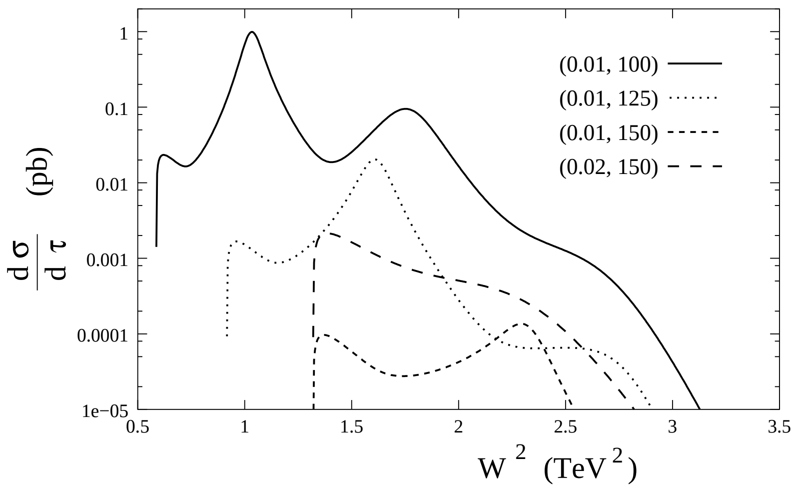

Figure 8: The invariant mass W^2 dependence of the two graviton production cross section in proton anti-proton collisions at $\sqrt{s} = 2\,\text{TeV}$ for several different values of the parameters (c_0, m_0), where m_0 is given in GeV. Here $W^2 = \tau s$ is the invariant mass of the two graviton final state.

production channel. The gravitons in the final state can be identified through their decay products which include a pair of jets, a lepton pair or a vector boson pair. We find that LHC can study a fairly large range of the parameter space of the RS model with this process. The process provides a nontrivial test of the RS model since its calculation requires the next to leading order Feynman rules.

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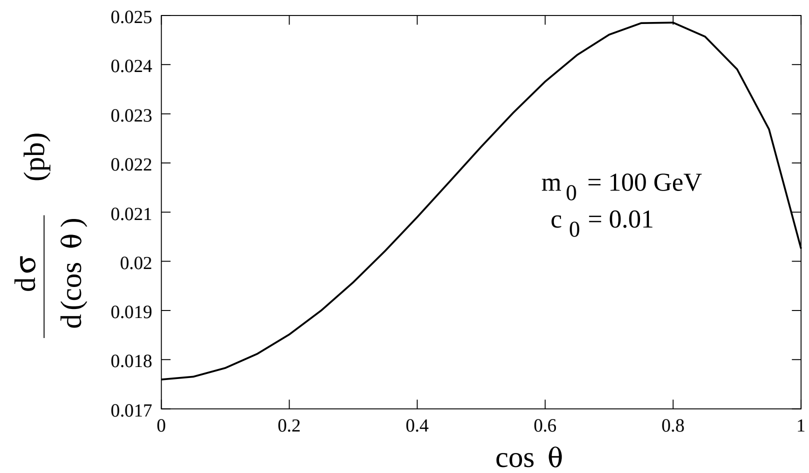

Figure 9: The $\cos\theta$ dependence of the two graviton production cross section in proton anti-proton collisions at $\sqrt{s} = 2\,\mathrm{TeV}$. Here θ is the center of mass scattering angle in the partonic subsystem.

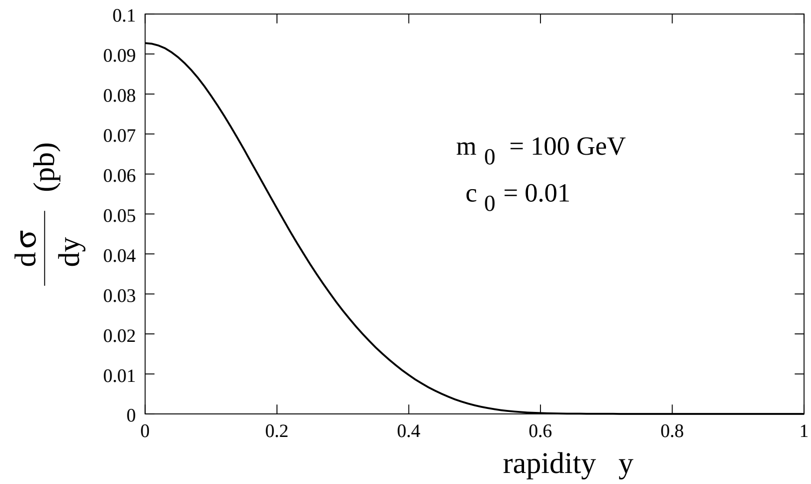

Figure 10: The rapidity dependence of the two graviton production cross section in proton anti-proton collisions at $\sqrt{s} = 2\,\mathrm{TeV}$.

A. Coupling to matter fields

A.1 Coupling to scalar bosons

For a general complex scalar field Φ,we have

$$\mathcal{L}_s(g) = g^{\rho\sigma} D_\rho \Phi^\dagger D_\sigma \Phi - m_\Phi^2 \Phi^\dagger \Phi \tag{A.1}$$

$$\left.\frac{\delta \mathcal{L}_s}{\delta \hat{g}^{\mu\nu}}\right|_{\hat{g}=\eta} = \frac{1}{2}(D_\mu \Phi^\dagger D_\nu \Phi + D_\nu \Phi^\dagger D_\mu \Phi) \tag{A.2}$$

$$\left.\frac{\delta^2 \mathcal{L}_s}{\delta \hat{g}^{\mu\nu} \delta \hat{g}^{\rho\sigma}}\right|_{\hat{g}=\eta} = 0\,, \tag{A.3}$$

where the gauge covariant derivative is defined as

$$D_\mu = \partial_\mu + igA^a_\mu T^a \tag{A.4}$$

with g the coupling, A^a_μ the gauge fields and T^a the Lie algebra generators.

Substituting the expressions as given in eq. (A.1)–(A.3) in the lagrangian given in eq. (2.17), we finally have the Feynman rule for the vertex involving two gravitons, and two scalars as follows:

$$\begin{aligned} h^{(n)}_{\mu\nu}h^{(m)}_{\rho\sigma}\Phi\Phi : i\frac{1}{\Lambda_\pi^2}[[&\eta_{\sigma\nu}(k_{1\mu}k_{2\rho}+k_{1\rho}k_{2\mu}) + \eta_{\mu\sigma}(k_{1\rho}k_{2\nu}+k_{1\nu}k_{2\rho}) + \eta_{\nu\rho}(k_{1\mu}k_{2\sigma}+k_{1\sigma}k_{2\mu})]+ \\ &+ [\eta_{\mu\rho}(k_{1\sigma}k_{2\nu}+k_{1\nu}k_{2\sigma})+\eta_{\mu\nu}(k_{1\rho}k_{2\sigma}+k_{1\sigma}k_{2\rho})-\eta_{\rho\sigma}(k_{1\mu}k_{2\nu}+k_{1\nu}k_{2\mu})]- \\ &- [(\eta_{\mu\rho}\eta_{\nu\sigma}+\eta_{\nu\rho}\eta_{\mu\sigma}-\eta_{\mu\nu}\eta_{\rho\sigma})(k_1.k_2 - m_\Phi^2)]] \,. \end{aligned} \tag{A.5}$$

This can be rearranged in the following simplified manner,

$$i\frac{1}{\Lambda_\pi^2}\left(C_{\mu\nu,\rho\sigma}m_\Phi^2 + C_{\mu\nu,\ \rho\sigma}|_{\lambda\eta}k_1^\lambda k_2^\eta\right), \tag{A.6}$$

where k_1, k_2 are the four-momenta of the scalars,

$$C_{\mu\nu,\rho\sigma} = \eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho} - \eta_{\mu\nu}\eta_{\rho\sigma}$$

and

$$C_{\mu\nu,\rho\sigma}|_{\lambda\eta} = \frac{1}{2}\left[\eta_{\mu\lambda}C_{\rho\sigma,\ \nu\eta} + \eta_{\sigma\lambda}C_{\mu\nu,\ \rho\eta} + \eta_{\rho\lambda}C_{\mu\nu,\ \sigma\eta} + \eta_{\nu\lambda}C_{\mu\eta,\ \rho\sigma} - \eta_{\lambda\eta}C_{\mu\nu,\ \rho\sigma} + (\lambda \leftrightarrow \eta)\right].$$

Similarly, one can obtain the Feynman rules for the vertices involving two gravitons, two scalars and one gauge bosons(and two gauge bosons) as follows:

$$h^{(n)}_{\mu\nu}h^{(m)}_{\rho\sigma}A^a_\lambda\Phi\Phi : -\frac{ig}{\Lambda_\pi^2}C_{\mu\nu,\ \rho\sigma}|_{\lambda\eta}(k_1+k_2)^\eta T^a_{MN} \tag{A.7}$$

$$h^{(n)}_{\mu\nu}h^{(m)}_{\rho\sigma}A^a_\eta A^b_\lambda\Phi\Phi : -\frac{ig^2}{\Lambda_\pi^2}C_{\mu\nu,\ \rho\sigma}|_{\lambda\eta}\{T^a, T^b\}_{MN}\,. \tag{A.8}$$

A.2 Couplings to fermions

Vierbein formalism is required in order to incorporate fermions in the gravitation theory. The Fermion lagrangian is given by

$$\mathcal{L}_\psi(\hat{g}) = \overline{\psi}i\gamma^\rho\mathcal{D}_\rho\psi - m_\psi\overline{\psi}\psi = \frac{1}{2}(\overline{\psi}i\gamma^\rho\mathcal{D}_\rho\psi - \mathcal{D}_\rho\overline{\psi}i\gamma^\rho\psi) - m_\psi\overline{\psi}\psi\,. \tag{A.9}$$

The covariant derivative $\mathcal{D}_\rho\psi$ is given by,

$$\mathcal{D}_\mu\psi = \left(D_\mu + \frac{1}{2}w^{ab}_\mu\sigma_{ab}\right)\psi\,. \tag{A.10}$$

The spin connection, w^{ab}_μ can be written as,

$$w_{\mu ab} = \frac{1}{2}\left(\partial_\mu e_{b\nu} - \partial_\nu e_{b\mu}\right)e^\nu_a - \frac{1}{2}\left(\partial_\mu e_{a\nu} - \partial_\nu e_{a\mu}\right)e^\nu_b - \frac{1}{2}e^\rho_a e^\sigma_b\left(\partial_\rho e_{c\sigma} - \partial_\sigma e_{c\rho}\right)e^c_\mu\,, \tag{A.11}$$

where $e^a_\mu e^b_\nu \eta_{ab} = g_{\mu\nu}$, $\gamma^\mu = e^\mu_a \gamma^a$, $e^\mu_a e_{b\mu} = \eta_{ab}$ and $\sigma_{ab} = \frac{1}{4}[\gamma_a, \gamma_b]$. The conserved energy-momentum tensor of the fermion field is given by,

$$\begin{aligned} T^\psi_{\mu\nu} =& -\eta_{\mu\nu}(\overline{\psi} i\gamma^\sigma D_\sigma \psi - m_\psi \overline{\psi}\psi) + \frac{i}{2}\left[(\overline{\psi}\gamma_\mu D_\nu \psi + \overline{\psi}\gamma_\nu D_\mu \psi)\right] + \frac{i}{2}\eta_{\mu\nu}\partial_\sigma(\overline{\psi}\gamma^\sigma\psi) - \\ & - \frac{i}{4}\left[\partial_\mu(\overline{\psi}\gamma_\nu\psi) + \partial_\nu(\overline{\psi}\gamma_\mu\psi)\right]. \end{aligned} \tag{A.12}$$

Now taking the functional derivative of $\mathcal{L}_\psi(\hat{g})$ successively, one finds that

$$\begin{aligned} \left.\frac{\delta^2 \mathcal{L}_\psi}{\delta \hat{g}^{\mu\nu}\delta\hat{g}^{\rho\sigma}}\right|_{\hat{g}=\eta} =& -\frac{1}{128}[\eta_{\mu\rho}\partial^\alpha(\overline{\psi}\epsilon_{\nu\sigma\alpha\tau}\gamma^\tau\gamma^5\psi) + \eta_{\mu\sigma}\partial^\alpha(\overline{\psi}\epsilon_{\nu\rho\alpha\tau}\gamma^\tau\gamma^5\psi) + \\ & + \eta_{\nu\sigma}\partial^\alpha(\overline{\psi}\epsilon_{\mu\rho\alpha\tau}\gamma^\tau\gamma^5\psi) + \eta_{\nu\rho}\partial^\alpha(\overline{\psi}\epsilon_{\mu\sigma\alpha\tau}\gamma^\tau\gamma^5\psi)]. \end{aligned} \tag{A.13}$$

After substituting eq. (A.9), (A.12) and (A.13) in the expression of $\mathcal{L}^{mn}_2$ (eq. (2.17)), we get the Feynman rule for the vertex involving two fermions and two gravitons as follows,

$$\begin{aligned} h^{(n)}_{\mu\nu}h^{(m)}_{\rho\sigma}\psi\psi : \frac{i}{4\Lambda_\pi^2}\delta_{MN}&[[\eta_{\sigma\mu}(\gamma_\rho(p_1+p_2)_\nu + \gamma_\nu(p_1+p_2)_\rho - 2\eta_{\rho\nu}(\not{p}_1 + \not{p}_2 - 2m_\psi))] + \\ & + [\eta_{\rho\mu}(\gamma_\sigma(p_1+p_2)_\nu + \gamma_\nu(p_1+p_2)_\sigma - 2\eta_{\sigma\nu}(\not{p}_1 + \not{p}_2 - 2m_\psi))] + \\ & + [\eta_{\nu\rho}(\gamma_\mu(p_1+p_2)_\sigma + \gamma_\sigma(p_1+p_2)_\mu - 2\eta_{\mu\sigma}(\not{p}_1 + \not{p}_2 - 2m_\psi))] + \\ & + [\eta_{\nu\sigma}(\gamma_\mu(p_1+p_2)_\rho + \gamma_\rho(p_1+p_2)_\mu - 2\eta_{\mu\rho}(\not{p}_1 + \not{p}_2 - 2m_\psi))] - \\ & - [\eta_{\mu\nu}(\gamma_\rho(p_1+p_2)_\sigma + \gamma_\sigma(p_1+p_2)_\rho - 2\eta_{\rho\sigma}(\not{p}_1 + \not{p}_2 - 2m_\psi))] - \\ & - [\eta_{\rho\sigma}(\gamma_\mu(p_1+p_2)_\nu + \gamma_\nu(p_1+p_2)_\mu - 2\eta_{\mu\nu}(\not{p}_1 + \not{p}_2 - 2m_\psi))] + \\ & + 2C_{\mu\nu,\ \rho\sigma}[(\not{p}_1 + \not{p}_2 - 2m_\psi)] + \\ & + \frac{1}{2}[\eta_{\mu\rho}\epsilon_{\nu\sigma\lambda\tau}\gamma^\tau\gamma^5(p_1-p_2)^\lambda + \eta_{\mu\sigma}\epsilon_{\nu\rho\lambda\tau}\gamma^\tau\gamma^5(p_1-p_2)^\lambda + \\ & + \eta_{\nu\sigma}\epsilon_{\mu\rho\lambda\tau}\gamma^\tau\gamma^5(p_1-p_2)^\lambda + \eta_{\nu\rho}\epsilon_{\mu\sigma\lambda\tau}\gamma^\tau\gamma^5(p_1-p_2)^\lambda]], \end{aligned} \tag{A.14}$$

where $\gamma^5 = \gamma^0\gamma^1\gamma^2\gamma^3$ and δ_{MN} is defined in the flavour basis (M, N are the flavour indices) and p_1, p_2 are the four momenta of the two fermions.

Similarly the Feynman rules for the vertices involving two gravitons, two fermions and one gauge particle is,

$$\begin{aligned} h^{(n)}_{\mu\nu}h^{(m)}_{\rho\sigma}A^a_\lambda\psi\psi : \frac{ig}{\Lambda_\pi^2}T^a_{MN}&[\gamma^\eta\left[\eta_{\nu\sigma}(C_{\mu\rho,\ \lambda\eta} - \eta_{\mu\rho}\eta_{\lambda\eta}) + (\nu \leftrightarrow \mu)\right] + \\ & + \gamma^\eta\left[\{\eta_{\nu\rho}(C_{\mu\sigma,\ \lambda\eta} - \eta_{\mu\sigma}\eta_{\lambda\eta}) + (\nu \leftrightarrow \mu)\}\right] + \\ & + \gamma^\eta\left[\eta_{\sigma\rho}(C_{\mu\nu,\ \lambda\eta} - \eta_{\mu\nu}\eta_{\lambda\eta}) + \eta_{\mu\nu}(C_{\rho\sigma,\ \lambda\eta} - \eta_{\rho\sigma}\eta_{\lambda\eta})\right] - \\ & - \gamma^\eta\left[\eta_{\lambda\eta}(C_{\mu\nu,\ \rho\sigma})\right]] \end{aligned} \tag{A.15}$$

A.3 Coupling to gauge fields

The lagrangian and conserved energy-momentum tensor for the gauge field are given by,

$$\mathcal{L}_A(\hat{g}) = -\frac{1}{4}F^{\tau\eta}F_{\tau\eta} + \frac{1}{2}{m_A}^2 A^\tau A_\tau - \frac{1}{2\xi}(\partial_\tau A^\tau + \Gamma^\tau_{\tau\lambda}A^\lambda)^2 \tag{A.16}$$

$$\begin{aligned} T_{\mu\nu} =& \eta_{\mu\nu}\left[\frac{1}{4}F^{\tau\eta}F_{\tau\eta} - \frac{1}{2}{m_A}^2 A^\tau A_\tau\right] - \left[F^\tau_\mu F_{\nu\tau} - m_A^2 A_\mu A_\nu\right] + \\ & + \frac{1}{\xi}\left[\partial_\mu\partial^\tau A_\tau A_\nu + \partial_\nu\partial^\tau A_\tau A_\mu\right] - -\frac{1}{\xi}\eta_{\mu\nu}\left[\partial_\eta\partial^\tau A_\tau A^\eta + \frac{1}{2}(\partial^\tau A_\tau)^2\right]. \end{aligned} \tag{A.17}$$

After successive functional differentiation, one obtain,

$$\begin{aligned}
\left.\frac{\delta^2\mathcal{L}_\psi}{\delta\hat{g}^{\mu\nu}\delta\hat{g}^{\rho\sigma}}\right|_{\hat{g}=\eta} &= -\frac{1}{4}\left[F_{\mu\sigma}F_{\nu\rho}+F_{\mu\rho}F_{\nu\sigma}\right]_{\hat{g}=\eta}+\frac{1}{4\xi}\left[\partial_\rho\partial_\mu A_\nu A_\sigma+\partial_\rho\partial_\nu A_\mu A_\sigma\right]+\\
&\quad+\frac{1}{4\xi}\left[\partial_\sigma\partial_\mu A_\nu A_\rho+\partial_\sigma\partial_\nu A_\mu A_\rho\right]-\frac{1}{4\xi}\eta_{\rho\sigma}\left[(\partial_\mu A_\nu+\partial_\nu A_\mu)\partial_\eta A^\eta\right]-\\
&\quad-\frac{1}{4\xi}\eta_{\rho\sigma}\left[(\partial_\eta\partial_\mu A_\nu A^\eta+\partial_\eta\partial_\nu A_\mu A^\eta)\right]+\\
&\quad+\frac{1}{4\xi}C_{\mu\nu,\rho\sigma}\left[(\partial_\tau\partial_\eta A^\eta A^\tau+(\partial_\tau A^\tau)^2\right]. \qquad\qquad (A.18)
\end{aligned}$$

The sets of eq. (A.16), (A.17) and (A.18) together when substituted in eq. (2.17), gives us the Feynman rule for the vertex involving two gravitons and two gauge fields as follows:

$$\begin{aligned}
&h^{(n)}_{\mu\nu}h^{(m)}_{\rho\sigma}A^a_\tau A^b_\eta:\\
&:\frac{i}{\Lambda_\pi^2}\delta^{ab}\Bigg[\left[\eta_{\sigma\mu}(C_{\rho\nu,\ \tau\eta}(k_1.k_2+m_A^2)+D_{\rho\nu,\ \tau\eta}(k_1,k_2)+\xi^{-1}E_{\rho\nu,\ \tau\eta})+(\rho\leftrightarrow\sigma)\right]+\\
&\qquad+\left[\eta_{\nu\rho}(C_{\sigma\mu,\ \tau\eta}(k_1.k_2+m_A^2)+D_{\sigma\mu,\ \tau\eta}(k_1,k_2)+\xi^{-1}E_{\sigma\mu,\ \tau\eta})+(\rho\leftrightarrow\sigma)\right]-\\
&\qquad-\left[\eta_{\mu\nu}(C_{\rho\sigma,\tau\eta}(k_1.k_2+m_A^2)+D_{\rho\sigma,\tau\eta}(k_1,k_2)+\xi^{-1}E_{\rho\sigma,\tau\eta})+(\mu\leftrightarrow\rho,\nu\leftrightarrow\sigma)\right]+\\
&\qquad+C_{\mu\nu,\ \rho\sigma}\left[(k_1.k_2+m_A^2)\eta_{\tau\eta}-(k_{1\eta}k_{2\tau}-\xi^{-1}k_{1\tau}k_{2\eta})\right]+\\
&\qquad+[(\eta_{\rho\tau}\eta_{\sigma\eta}k_{1\nu}k_{2\mu}+\eta_{\rho\eta}\eta_{\tau\sigma}k_{1\mu}k_{2\nu})-(\eta_{\rho\tau}\eta_{\mu\eta}k_{1\nu}k_{2\sigma}+\eta_{\rho\eta}\eta_{\tau\mu}k_{1\sigma}k_{2\nu})+(\rho\leftrightarrow\sigma)]+\\
&\qquad+[(\eta_{\nu\tau}\eta_{\mu\eta}k_{1\rho}k_{2\sigma}+\eta_{\nu\eta}\eta_{\tau\mu}k_{1\sigma}k_{2\rho})-(\eta_{\nu\tau}\eta_{\sigma\eta}k_{1\rho}k_{2\mu}+\eta_{\nu\eta}\eta_{\tau\sigma}k_{1\mu}k_{2\rho})+(\rho\leftrightarrow\sigma)]-\\
&\qquad-\frac{1}{\xi}\left[\eta_{\nu\tau}\eta_{\sigma\eta}(k_{1\mu}k_{1\rho}+k_{2\mu}k_{2\rho})+(\mu\leftrightarrow\nu)\right]-\\
&\qquad-\frac{1}{\xi}\left[\eta_{\nu\tau}\eta_{\rho\eta}(k_{1\mu}k_{1\sigma}+k_{2\mu}k_{2\sigma})+(\mu\leftrightarrow\nu)\right]+\\
&\qquad+\frac{1}{\xi}\left[\eta_{\nu\tau}\eta_{\rho\sigma}(k_{1\eta}k_{2\mu}+k_{1\mu}k_{2\eta})+(\mu\leftrightarrow\nu)\right]+\\
&\qquad+\frac{1}{\xi}\left[\eta_{\nu\tau}\eta_{\rho\sigma}(k_{1\eta}k_{1\mu}+k_{2\mu}k_{2\eta})+(\mu\leftrightarrow\nu)\right]-\\
&\qquad-\frac{1}{\xi}\left[C_{\mu\nu,\ \rho\sigma}(k_{1\eta}k_{1\tau}+k_{2\tau}k_{2\eta})+(\mu\leftrightarrow\nu)\right]-\frac{1}{\xi}\left[2C_{\mu\nu,\ \rho\sigma}(k_{1\tau}k_{2\eta})\right]\Bigg], \qquad (A.19)
\end{aligned}$$

where k_1 and k_2 are the four-momenta of the gauge fields A_τ and A_η.

Proceeding in the same way, one also finds the Feynman rules for the vertices involving two gravitons, three gauge bosons and two gravitons, four gauge bosons

$$\left[h^{(n)}_{\mu\nu}h^{(m)}_{\rho\sigma}A^a_\lambda(k_1)A^b_\eta(k_2)A^c_\delta(k_3)\right]$$

as follows,

$$\begin{aligned}
&-\frac{ig}{\Lambda_\pi^2}f^{abc}[[\{\eta_{\sigma\nu}C_{\mu\rho,\ \lambda\eta}(k_1-k_2)_\delta+\eta_{\sigma\nu}C_{\mu\rho,\ \lambda\delta}(k_3-k_1)_\eta+\\
&\qquad+\eta_{\sigma\nu}C_{\mu\rho,\ \eta\delta}(k_2-k_3)_\lambda+(\mu\leftrightarrow\nu)\}+\{(\rho\leftrightarrow\sigma)\}]+\\
&\qquad+\{\eta_{\sigma\nu}F_{\mu\rho,\ \lambda\eta\delta}(k_1,k_2,k_3)+\eta_{\sigma\mu}F_{\nu\rho,\ \lambda\eta\delta}(k_1,k_2,k_3)+(\rho\leftrightarrow\sigma)\}-
\end{aligned}$$

$$
\begin{aligned}
&- \eta_{\rho\sigma} C_{\mu\nu,\ \lambda\eta}(k_1 - k_2)_\delta - \eta_{\rho\sigma} C_{\mu\nu,\ \lambda\delta}(k_3 - k_1)_\eta - \eta_{\rho\sigma} C_{\mu\nu,\ \eta\delta}(k_2 - k_3)_\lambda - \\
&- \eta_{\mu\nu} C_{\rho\sigma,\ \lambda\eta}(k_1 - k_2)_\delta - \eta_{\mu\nu} C_{\rho\sigma,\ \lambda\delta}(k_3 - k_1)_\eta - \eta_{\mu\nu} C_{\rho\sigma,\ \eta\delta}(k_2 - k_3)_\lambda - \\
&- \eta_{\rho\sigma} F_{\mu\nu,\ \lambda\eta\delta}(k_1, k_2, k_3) - \eta_{\mu\nu} F_{\rho\sigma,\ \lambda\eta\delta}(k_1, k_2, k_3) + \\
&+ C_{\mu\nu,\ \rho\sigma}(\eta_{\lambda\eta}(k_1 - k_2)_\delta + \eta_{\eta\delta}(k_2 - k_3)_\lambda + \eta_{\lambda\delta}(k_3 - k_1)_\eta) - \\
&- [\{(k_{3\mu}\eta_{\delta\sigma} - k_{3\sigma}\eta_{\mu\delta})(\eta_{\rho\eta}\eta_{\nu\lambda} - \eta_{\nu\eta}\eta_{\rho\lambda}) \\
&\quad + \eta_{\nu\delta}(\eta_{\sigma\eta}\eta_{\rho\lambda}k_{2\mu} - \eta_{\rho\eta}\eta_{\sigma\lambda}k_{1\mu} + \eta_{\rho\eta}\eta_{\mu\lambda}k_{1\sigma} - \eta_{\rho\lambda}\eta_{\mu\eta}k_{2\sigma}) + \\
&\quad + \eta_{\rho\delta}(\eta_{\sigma\lambda}\eta_{\nu\eta}k_{2\mu} - \eta_{\sigma\eta}\eta_{\nu\lambda}k_{1\mu} + \eta_{\mu\eta}\eta_{\nu\lambda}k_{1\sigma} - \eta_{\mu\lambda}\eta_{\nu\eta}k_{2\sigma}) + (\rho \leftrightarrow \sigma)\} - \\
&\quad - (\nu \leftrightarrow \mu, \rho \leftrightarrow \sigma)]] \qquad \text{(A.20)}
\end{aligned}
$$

Finally the Feynman rule for the vertex involving two gravitons and four gauge bosons

$$\left[h^{(n)}_{\mu\nu} h^{(m)}_{\rho\sigma} A^a_\lambda(k_1) A^b_\eta(k_2) A^c_\delta(k_3) A^d_\tau(k_4)\right]$$

is given by,

$$
\begin{aligned}
&\frac{ig^2}{\Lambda_\pi^2}\Big[\{\eta_{\sigma\nu}(f^{eac}f^{ebd}G_{\mu\rho,\ \lambda\eta\delta\tau} + f^{eab}f^{ecd}G_{\mu\rho,\ \lambda\delta\eta\tau} + f^{ead}f^{ebc}G_{\mu\rho,\ \lambda\eta\tau\delta}) + (\mu \leftrightarrow \nu)\} + \\
&\quad + \{\eta_{\nu\rho}(f^{eac}f^{ebd}G_{\mu\sigma,\ \lambda\eta\delta\tau} + f^{eab}f^{ecd}G_{\mu\sigma,\ \lambda\delta\eta\tau} + f^{ead}f^{ebc}G_{\mu\sigma,\ \lambda\eta\tau\delta}) + (\mu \leftrightarrow \nu)\} - \\
&\quad - \eta_{\rho\sigma}(f^{eac}f^{ebd}G_{\mu\nu,\ \lambda\eta\delta\tau} + f^{eab}f^{ecd}G_{\mu\nu,\ \lambda\delta\eta\tau} + f^{ead}f^{ebc}G_{\mu\nu,\ \lambda\eta\tau\delta}) - \\
&\quad - \eta_{\mu\nu}(f^{eac}f^{ebd}G_{\rho\sigma,\ \lambda\eta\delta\tau} + f^{eab}f^{ecd}G_{\rho\sigma,\ \lambda\delta\eta\tau} + f^{ead}f^{ebc}G_{\rho\sigma,\ \lambda\eta\tau\delta}) - \\
&\quad - C_{\mu\nu,\ \rho\sigma}\{f^{eac}f^{ebd}(\eta_{\lambda\eta}\eta_{\delta\tau} - \eta_{\lambda\tau}\eta_{\delta\eta}) + f^{eab}f^{ecd}(\eta_{\lambda\delta}\eta_{\eta\tau} - \eta_{\lambda\tau}\eta_{\delta\eta}) + \\
&\qquad\qquad + f^{ead}f^{ebc}(\eta_{\lambda\eta}\eta_{\tau\delta} - \eta_{\lambda\delta}\eta_{\tau\eta})\} - \\
&\quad - [\{f^{eab}f^{ecd}(\eta_{\nu\eta}\eta_{\sigma\lambda}\eta_{\rho\delta} - \eta_{\rho\delta}\eta_{\nu\lambda}\eta_{\sigma\eta}) + f^{eca}f^{ebd}(\eta_{\rho\eta}\eta_{\nu\lambda}\eta_{\delta\sigma} - \eta_{\rho\eta}\eta_{\nu\delta}\eta_{\sigma\lambda}) + \\
&\qquad + f^{ebc}f^{eda}(\eta_{\rho\lambda}\eta_{\nu\eta}\eta_{\delta\sigma} - \eta_{\rho\lambda}\eta_{\sigma\eta}\eta_{\nu\delta})\}\eta_{\mu\tau} + \{f^{eab}f^{ecd}(\eta_{\mu\delta}\eta_{\nu\lambda}\eta_{\sigma\eta} - \eta_{\mu\delta}\eta_{\nu\eta}\eta_{\sigma\lambda}) + \\
&\qquad + f^{eca}f^{ebd}(\eta_{\nu\delta}\eta_{\mu\eta}\eta_{\sigma\lambda} - \eta_{\mu\eta}\eta_{\nu\lambda}\eta_{\delta\sigma}) + f^{ebc}f^{eda}(\eta_{\mu\lambda}\eta_{\nu\eta}\eta_{\delta\sigma} - \eta_{\mu\lambda}\eta_{\sigma\eta}\eta_{\nu\delta})\}\eta_{\rho\tau} + \\
&\qquad + \{f^{eab}f^{ecd}(\eta_{\delta\sigma}\eta_{\mu\eta}\eta_{\rho\lambda} - \eta_{\delta\sigma}\eta_{\mu\lambda}\eta_{\rho\eta}) + f^{eca}f^{ebd}(\eta_{\rho\delta}\eta_{\mu\lambda}\eta_{\sigma\eta} - \eta_{\mu\delta}\eta_{\rho\lambda}\eta_{\sigma\eta}) + \\
&\qquad\quad + f^{ebc}f^{eda}(\eta_{\rho\delta}\eta_{\mu\eta}\eta_{\sigma\lambda} - \eta_{\mu\delta}\eta_{\rho\eta}\eta_{\sigma\lambda})\}\eta_{\nu\tau} + \\
&\qquad + \{f^{eab}f^{ecd}(\eta_{\nu\delta}\eta_{\mu\lambda}\eta_{\rho\eta} - \eta_{\nu\delta}\eta_{\mu\eta}\eta_{\rho\lambda}) + f^{eca}f^{ebd}(\eta_{\mu\delta}\eta_{\rho\lambda}\eta_{\nu\eta} - \eta_{\rho\delta}\eta_{\mu\lambda}\eta_{\nu\eta}) + \\
&\qquad\quad + f^{ebc}f^{eda}(\eta_{\mu\delta}\eta_{\rho\eta}\eta_{\nu\lambda} - \eta_{\rho\delta}\eta_{\mu\eta}\eta_{\nu\lambda})\}\eta_{\tau\delta} + \{\rho \leftrightarrow \sigma\}]\Big], \qquad \text{(A.21)}
\end{aligned}
$$

where,

$$
\begin{aligned}
D_{\mu\nu,\ \rho\sigma(k_1,k_2)} &= \eta_{\mu\nu}k_{1\sigma}k_{2\rho} - [\eta_{\mu\sigma}k_{1\nu}k_{2\rho} + \eta_{\mu\rho}k_{1\sigma}k_{2\nu} - \eta_{\rho\sigma}k_{1\mu}k_{2\nu} + (\mu \leftrightarrow \nu)] \\
E_{\mu\nu,\ \rho\sigma(k_1,k_2)} &= \eta_{\mu\nu}(k_{1\rho}k_{1\sigma} + k_{2\rho}k_{2\sigma} + k_{1\rho}k_{2\sigma}) - [\eta_{\nu\sigma}k_{1\mu}k_{1\rho} + \eta_{\nu\rho}k_{2\mu}k_{2\sigma} + (\mu \leftrightarrow \nu)] \\
F_{\mu\nu,\ \rho\sigma\lambda(k_1,k_2,k_3)} &= \eta_{\mu\rho}\eta_{\sigma\lambda}(k_2 - k_3)_\nu + \eta_{\mu\sigma}\eta_{\rho\lambda}(k_3 - k_1)_\nu + \eta_{\mu\lambda}\eta_{\rho\sigma}(k_1 - k_2)_\nu + (\mu \leftrightarrow \nu) \\
G_{\mu\nu,\ \rho\sigma\lambda\delta} &= \eta_{\mu\nu}(\eta_{\rho\sigma}\eta_{\lambda\delta} - \eta_{\rho\delta}\eta_{\sigma\lambda}) + \\
&\quad + [\eta_{\mu\rho}\eta_{\nu\delta}\eta_{\lambda\sigma} + \eta_{\mu\lambda}\eta_{\nu\sigma}\eta_{\rho\delta} - \eta_{\mu\rho}\eta_{\nu\sigma}\eta_{\lambda\delta} - \eta_{\mu\lambda}\eta_{\nu\delta}\eta_{\rho\sigma}] + [\mu \leftrightarrow \nu].
\end{aligned}
$$

B. Triple graviton vertex

To find the feynman rules for three graviton vertex we expand the pure gravity lagrangian upto next order in weak field approximation. In this approximation

$$\begin{aligned} L_g &= \frac{1}{\kappa^2}\sqrt{g}R = L_0 + \kappa L_1 + \cdots \\ &= \frac{1}{4}\left(\partial^\mu h^{\nu\rho}\partial_\mu h_{\nu\rho} - \partial^\mu h^\nu_\nu \partial_\mu h^\rho_\rho - \partial^\mu h^\nu_\mu \partial_\rho h^{\rho\nu} + 2\partial_\mu h^{\mu\nu}\partial_\nu h^\rho_\rho\right) - \\ &- \frac{\kappa}{4}\left[h^{\mu\nu}\partial_\mu h^{\alpha\beta}\partial_\nu h_{\alpha\beta} - \frac{1}{2}h^{\mu\nu}\partial_\mu h^{\alpha\alpha}\partial_\nu h_{\beta\beta}\right] - \\ &- \frac{\kappa}{4}\left[2h^{\mu\nu}\partial^\beta h_{\mu\alpha}\partial^\alpha h_{\nu\beta} + h^{\mu\nu}\partial_\alpha h^{\beta\beta}\partial^\alpha h_{\mu\nu} - 2h^{\mu\nu}\partial_\alpha h_{\mu\beta}\partial^\alpha h^\beta_\nu\right] + \cdots . \end{aligned} \tag{B.1}$$

From the expression given in L_1, feynman rule for three graviton vertex reads

$$\begin{aligned} &\frac{\kappa}{8}\left[(k_{2\mu}k_{3\nu} + k_{2\nu}k_{3\mu})F_{\rho\lambda k\sigma} + (k_{1\rho}k_{3\sigma} + k_{1\sigma}k_{3\rho})F_{\mu\lambda k\nu} + (k_{1\lambda}k_{2k} + k_{1k}k_{2\lambda})F_{\mu\rho\sigma\nu}\right] - \\ &- \frac{\kappa}{8}\left[(k_{2\mu}k_{3\nu} + k_{2\nu}k_{3\mu})\eta_{\lambda k}\eta_{\rho\sigma} + \eta_{\mu\nu}\eta_{\lambda k}(k_{1\rho}k_{3\sigma} + k_{1\sigma}k_{3\rho})\right] - \\ &- \frac{\kappa}{8}\left[\eta_{\mu\nu}\eta_{\rho\sigma}(k_{1\lambda}k_{2k} + k_{1k}k_{2\lambda})\right] + \\ &+ \frac{\kappa}{8}\left[F_{\mu\rho\lambda\nu}k_{2k}k_{3\sigma} + F_{\mu\rho k\nu}k_{2\lambda}k_{3\sigma} + F_{\mu\sigma\lambda\nu}k_{2k}k_{3\rho} + F_{\mu\sigma k\nu}k_{2\lambda}k_{3\rho}\right] + \\ &+ \frac{\kappa}{8}\left[F_{\mu\rho\sigma\lambda}k_{1k}k_{3\nu} + F_{\mu\rho\sigma k}k_{1\lambda}k_{3\nu} + F_{\nu\rho\sigma\lambda}k_{1k}k_{3\mu} + F_{\nu\rho\sigma k}k_{1\lambda}k_{3\mu}\right] + \\ &+ \frac{\kappa}{8}\left[F_{\mu\lambda k\rho}k_{1\sigma}k_{2\nu} + F_{\mu\lambda k\sigma}k_{1\rho}k_{2\nu} + F_{\nu\lambda k\rho}k_{1\sigma}k_{2\nu} + F_{\nu\lambda k\sigma}k_{1\rho}k_{2\mu}\right] + \\ &+ \frac{\kappa}{8}\left[G_{\mu\nu\rho\lambda\sigma k}k_1.k_2 + G_{\rho\sigma\mu\lambda\nu k}k_2.k_3 + G_{\mu\nu\lambda\rho k\sigma}k_1.k_3\right] - \\ &- \frac{\kappa}{8}\left[I_{\mu\nu\rho\sigma\lambda k}(k_1.k_2 + k_2.k_3 + k_3.k_1)\right], \end{aligned} \tag{B.2}$$

where F, G and I are defined by the following relations

$$F_{\mu\nu\rho\sigma} = \eta_{\mu\nu}\eta_{\rho\sigma} + \eta_{\mu\rho}\eta_{\nu\sigma} \tag{B.3}$$

$$G_{\mu\nu\rho\lambda\sigma k} = \eta_{\mu\nu}F_{\rho\lambda k\sigma} + \eta_{\rho\sigma}F_{\mu\lambda k\nu} \tag{B.4}$$

$$I_{\mu\nu\rho\sigma\lambda k} = \eta_{\mu\rho}F_{\nu\lambda k\sigma} + \eta_{\mu\sigma}F_{\nu\lambda k\rho} + \eta_{\nu\rho}F_{\mu\lambda k\sigma} + \eta_{\nu\sigma}F_{\mu\lambda k\rho} \tag{B.5}$$

C. Cross section for two graviton production through fermion anti-fermion annihilation

In this appendix we give the detailed form of the cross section for two graviton production by fermion anti-fermion annihilation. The differential cross section for this process is given by.

$$\frac{d\sigma}{d(cos\theta)} = \frac{1}{32\pi s}\sqrt{1 - \frac{4M_n^2}{s}}(T_{11} + T_{22} + T_{33} + T_{12} + T_{23} + T_{13} + T_{44} + T_{14} + T_{24} + T_{34}), \tag{C.1}$$

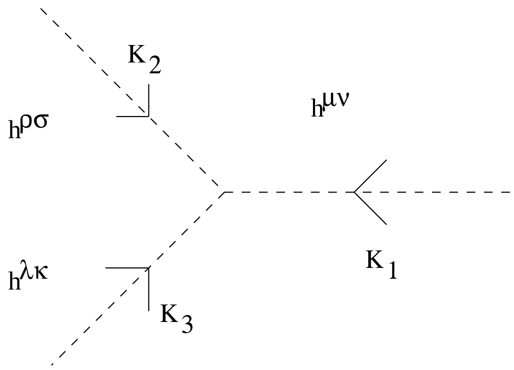

Figure 11: A typical triple graviton vertex.

where the terms T_{ij} inside bracket are given by,

$$T_{11} = \frac{\pi^2 c_0^4}{m_0^4}\left(\frac{-2\left({M_n}^2 - t\right)^4\left(9{M_n}^8 - 4t^3u + 12{M_n}^2t^2\left(t+u\right) - {M_n}^4t\left(20t+9u\right)\right)}{9{M_n}^8t^2}\right) \qquad \text{(C.2)}$$

$$T_{22} = \frac{\pi^2 c_0^4}{m_0^4}\left(\frac{-2\left({M_n}^2 - u\right)^4\left(9{M_n}^8 - 4tu^3 + 12{M_n}^2u^2\left(t+u\right) - {M_n}^4u\left(9t+20u\right)\right)}{9{M_n}^8u^2}\right) \qquad \text{(C.3)}$$

$$\begin{aligned}
T_{33} = {} & \frac{\pi^2 c_0^4}{m_0^4}\left(\frac{2\left(1092{M_n}^{12} - 1230{M_n}^{10}\left(t+u\right)\right) + 16t(t-u)^2u(t+u)^2}{9{M_n}^8}\right) - \\
& - \frac{\pi^2 c_0^4}{m_0^4}\left(\frac{7{M_n}^6\left(t+u\right)\left(25t^2+52tu+25u^2\right)}{9{M_n}^8}\right) + \\
& + \frac{\pi^2 c_0^4}{m_0^4}\left(\frac{4{M_n}^8\left(113t^2+299tu+113u^2\right)}{9{M_n}^8}\right) - \\
& - \frac{\pi^2 c_0^4}{m_0^4}\left(\frac{{M_n}^2\left(t+u\right)\left(15t^4+62t^3u-34t^2u^2+62tu^3+15u^4\right)}{9{M_n}^8}\right) + \\
& + \frac{\pi^2 c_0^4}{m_0^4}\left(\frac{{M_n}^4\left(56t^4+370t^3u+84t^2u^2+370tu^3+56u^4\right)}{9{M_n}^8}\right)
\end{aligned} \qquad \text{(C.4)}$$

$$\begin{aligned}
T_{12} = {} & \frac{\pi^2 c_0^4}{m_0^4}\left(\frac{\left(2{M_n}^2 - t - u\right)\left(88{M_n}^{14} - 62{M_n}^{12}\left(t+u\right) - 4t^3u^3\left(t+u\right)\right)}{9M_n^8tu}\right) - \\
& - \frac{\pi^2 c_0^4}{m_0^4}\left(\frac{\left(2{M_n}^2 - t - u\right)\left(10{M_n}^{10}\left(7t^2+19tu+7u^2\right)\right)}{9M_n^8tu}\right) - \\
& - \frac{\pi^2 c_0^4}{m_0^4}\left(\frac{\left(2{M_n}^2 - t - u\right)2{M_n}^2t^2u^2\left(9t^2+14tu+9u^2\right)}{9M_n^8tu}\right) - \\
& - \frac{\pi^2 c_0^4}{m_0^4}\left(\frac{\left(2{M_n}^2 - t - u\right)\left({M_n}^4tu\left(t+u\right)\left(26t^2+47tu+26u^2\right)\right)}{9M_n^8tu}\right) +
\end{aligned}$$

$$+\frac{\pi^2 c_0^4}{m_0^4}\left(\frac{\left(2{M_n}^2-t-u\right)\left({M_n}^8\left(t+u\right)\left(62t^2+41tu+62u^2\right)\right)}{9M_n^8 tu}\right)+$$

$$+\frac{\pi^2 c_0^4}{m_0^4}\left(\frac{\left(2{M_n}^2-t-u\right)\left({M_n}^6\left(-12t^4+64t^3u+202t^2u^2+64tu^3-12u^4\right)\right)}{9{M_n}^8 tu}\right) \quad (C.5)$$

$$T_{13}=\frac{\pi^2 c_0^4}{m_0^4}\left(\frac{8\left(-99{M_n}^{14}+2t^4\left(t-u\right)u\left(t+u\right)+{M_n}^{12}\left(96t+63u\right)\right)}{9{M_n}^8 t}\right)+$$

$$+\frac{\pi^2 c_0^4}{m_0^4}\left(\frac{8\left({M_n}^{10}\left(5t^2+70tu-9u^2\right)-{M_n}^8 t\left(3t^2+92tu+55u^2\right)\right)}{9{M_n}^8 t}\right)-$$

$$-\frac{\pi^2 c_0^4}{m_0^4}\left(\frac{8\left({M_n}^2t^3\left(3t^3+12t^2u-5tu^2+2u^3\right)+{M_n}^6 t\left(-18t^3-40t^2u+tu^2+6u^3\right)\right)}{9{M_n}^8 t}\right)+$$

$$+\frac{\pi^2 c_0^4}{m_0^4}\left(\frac{8\left({M_n}^4t^2\left(13t^3+36t^2u+31tu^2+7u^3\right)\right)}{9{M_n}^8 t}\right) \quad (C.6)$$

$$T_{23}=\frac{\pi^2 c_0^4}{m_0^4}\left(\frac{8\left(-99{M_n}^{14}-2t\left(t-u\right)u^4\left(t+u\right)+{M_n}^{12}\left(63t+96u\right)\right)}{9{M_n}^8 u}\right)-$$

$$-\frac{\pi^2 c_0^4}{m_0^4}\left(\frac{8\left({M_n}^8 u\left(55t^2+92tu+3u^2\right)+{M_n}^{10}\left(-9t^2+70tu+5u^2\right)\right)}{9{M_n}^8 u}\right)+$$

$$+\frac{\pi^2 c_0^4}{m_0^4}\left(\frac{8\left({M_n}^6 u\left(6t^3+t^2u-40tu^2-18u^3\right)-{M_n}^2u^3\left(2t^3-5t^2u+12tu^2+3u^3\right)\right)}{9{M_n}^8 u}\right)+$$

$$+\frac{\pi^2 c_0^4}{m_0^4}\left(\frac{8\left({M_n}^4u^2\left(7t^3+31t^2u+36tu^2+13u^3\right)\right)}{9{M_n}^8 u}\right) \quad (C.7)$$

$$T_{44}=\frac{\lambda_s^2 c_0^4}{512m_0^8}\left(\frac{8688{M_n}^{16}-9720{M_n}^{14}\left(t+u\right)+9t(t-u)^2u(t+u)^4}{9{M_n}^8}\right)+$$

$$+\frac{\lambda_s^2 c_0^4}{512m_0^8}\left(\frac{20{M_n}^{10}\left(t+u\right)\left(85t^2+52tu+85u^2\right)}{9{M_n}^8}\right)+$$

$$+\frac{\lambda_s^2 c_0^4}{512m_0^8}\left(\frac{{M_n}^{12}\left(428t^2+5048tu+428u^2\right)}{9{M_n}^8}\right)-$$

$$-\frac{\lambda_s^2 c_0^4}{512m_0^8}\left(\frac{12{M_n}^2(t+u)^3\left(t^4+8t^3u-10t^2u^2+8tu^3+u^4\right)}{9{M_n}^8}\right)+$$

$$+\frac{\lambda_s^2 c_0^4}{512m_0^8}\left(\frac{4{M_n}^8\left(17t^4+403t^3u+24t^2u^2+403tu^3+17u^4\right)}{9{M_n}^8}\right)-$$

$$-\frac{\lambda_s^2 c_0^4}{512m_0^8}\left(\frac{4{M_n}^6\left(t+u\right)\left(117t^4+482t^3u+410t^2u^2+482tu^3+117u^4\right)}{9{M_n}^8}\right)+$$

$$+\frac{\lambda_s^2 c_0^4}{512m_0^8}\left(\frac{{M_n}^4(t+u)^2\left(135t^4+604t^3u+58t^2u^2+604tu^3+135u^4\right)}{9{M_n}^8}\right) \quad (C.8)$$

$$
\begin{aligned}
T_{14} = & -\frac{\pi c_0^4\lambda_s}{4m_0^6}\left(\frac{864{M_n}^{16}+3t^4\,(t-u)\,u(t+u)^2}{9{M_n}^8 t}\right)+\\
& +\frac{\pi c_0^4\lambda_s}{4m_0^6}\left(\frac{2{M_n}^2 t^4\,(3t-u)\,(t+u)\,(t+5u)}{9{M_n}^8 t}\right)+\\
& +\frac{\pi c_0^4\lambda_s}{4m_0^6}\left(\frac{12{M_n}^{14}\,(80t+63u)+{M_n}^{12}\left(-79t^2-404tu+207u^2\right)}{9{M_n}^8 t}\right)-\\
& -\frac{\pi c_0^4\lambda_s}{4m_0^6}\left(\frac{{M_n}^8 t\left(39t^3-49t^2u-323tu^2-171u^3\right)}{9{M_n}^8 t}\right)-\\
& -\frac{\pi c_0^4\lambda_s}{4m_0^6}\left(\frac{2{M_n}^{10}\left(153t^3+593t^2u+331tu^2-9u^3\right)}{9{M_n}^8 t}\right)+\\
& +\frac{\pi c_0^4\lambda_s}{4m_0^6}\left(\frac{2{M_n}^6 t\left(67t^4+201t^3u+175t^2u^2+37tu^3-6u^4\right)}{9{M_n}^8 t}\right)-\\
& -\frac{\pi c_0^4\lambda_s}{4m_0^6}\left(\frac{{M_n}^4 t^2\left(51t^4+186t^3u+143t^2u^2+94tu^3+18u^4\right)}{9{M_n}^8 t}\right)
\end{aligned}
\tag{C.9}
$$

$$
\begin{aligned}
T_{24} = & -\frac{\pi c_0^4\lambda_s}{4m_0^6}\left(\frac{\left(864{M_n}^{16}-3t\,(t-u)\,u^4(t+u)^2\right)}{9{M_n}^8 u}\right)+\\
& +\frac{\pi c_0^4\lambda_s}{4m_0^6}\left(\frac{2{M_n}^2\,(t-3u)\,u^4\,(t+u)\,(5t+u)}{9{M_n}^8 u}\right)+\\
& +\frac{\pi c_0^4\lambda_s}{4m_0^6}\left(\frac{12{M_n}^{14}\,(63t+80u)+{M_n}^{12}\left(207t^2-404tu-79u^2\right)}{9{M_n}^8 u}\right)-\\
& -\frac{\pi c_0^4\lambda_s}{4m_0^6}\left(\frac{{M_n}^8 u\left(-171t^3-323t^2u-49tu^2+39u^3\right)}{9{M_n}^8 u}\right)-\\
& -\frac{\pi c_0^4\lambda_s}{4m_0^6}\left(\frac{{M_n}^{10}\left(-18t^3+662t^2u+1186tu^2+306u^3\right)}{9{M_n}^8 u}\right)-\\
& -\frac{\pi c_0^4\lambda_s}{4m_0^6}\left(\frac{{M_n}^4 u^2\left(18t^4+94t^3u+143t^2u^2+186tu^3+51u^4\right)}{9{M_n}^8 u}\right)+\\
& +\frac{\pi c_0^4\lambda_s}{4m_0^6}\left(\frac{2{M_n}^6 u\left(-6t^4+37t^3u+175t^2u^2+201tu^3+67u^4\right))}{9{M_n}^8 u}\right)
\end{aligned}
\tag{C.10}
$$

$$
\begin{aligned}
T_{34} = & \frac{\pi c_0^4\lambda_s}{32m_0^6}\left(\frac{16\left(-588{M_n}^{14}+630{M_n}^{12}\,(t+u)\right)}{9{M_n}^8}\right)+\\
& +\frac{\pi c_0^4\lambda_s}{32m_0^6}\left(\frac{3t(t-u)^2u(t+u)^3}{9{M_n}^8}\right)-\\
& -\frac{\pi c_0^4\lambda_s}{32m_0^6}\left(\frac{{M_n}^8\,(t+u)\left(5t^2-124tu+5u^2\right)}{9{M_n}^8}\right)-
\end{aligned}
$$

$$
\begin{aligned}
&-\frac{\pi c_0^4\lambda_s}{32m_0^6}\left(\frac{2{M_n}^{10}\left(43t^2+244tu+43u^2\right)}{9{M_n}^8}\right)-\\
&-\frac{\pi c_0^4\lambda_s}{32m_0^6}\left(\frac{{M_n}^2(t+u)^2\left(3t^4+22t^3u-26t^2u^2+22tu^3+3u^4\right)}{9{M_n}^8}\right)-\\
&-\frac{\pi c_0^4\lambda_s}{32m_0^6}\left(\frac{4{M_n}^6\left(8t^4+61t^3u+30t^2u^2+61tu^3+8u^4\right)}{9{M_n}^8}\right)+\\
&+\frac{\pi c_0^4\lambda_s}{32m_0^6}\left(\frac{{M_n}^4\,(t+u)\left(21t^4+92t^3u+38t^2u^2+92tu^3+21u^4\right)}{9{M_n}^8}\right). \qquad \text{(C.11)}
\end{aligned}
$$

Here s, t and u are the Mandelstam variables.

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